# Applied Calculus — Quiz 6 — Key

(Attach a sheet of paper showing your work if needed.)

1. Find all relative extrema of $f(x) = x^3 + 15x^2$. Use the first or second derivative test to determine the nature of each extremum.

   $f'(x) = 3x^2 + 30x = 3x(x+10) \implies$ critical values are $0, -10$.

   $f''(x) = 6x + 30 \implies f''(0) = 30 > 0,\ f''(-10) = -60 + 30 < 0.$

   By the second derivative test,

   $f(0) = 0$ is a relative minimum value of $f$ at $x = 0$.

   $f(-10) = (-10)^3 + 15(-10)^2 = 500$ is a relative maximum value of $f$ at $x = -10$.

2. Sketch a graph of $y = f(x)$, a polynomial function satisfying all of the following:

   $f(-4) = 3,\ f'(-4) = 0,\ f''(-4) < 0,$

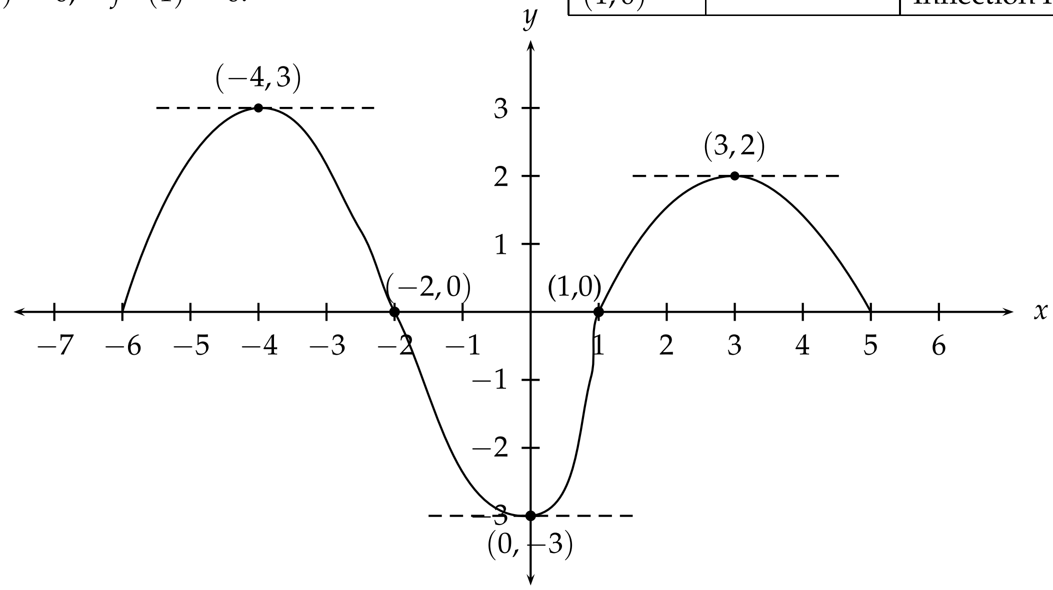
   $f(0) = -3, \quad f'(0) = 0, \quad f''(0) > 0,$

   $f(3) = 2, \quad f'(3) = 0, \quad f''(3) < 0,$

   $f(-2) = 0, \quad f''(-2) = 0,$

   $f(1) = 0, \quad f''(1) = 0.$

| $(x, f(x))$ | $f'$ | $f''$ |
|---|---|---|
| Point | Tangent Line | Concavity |
| $(-4, 3)$ | Horizontal | *Down* |
| $(0, -3)$ | Horizontal | *Up* |
| $(3, 2)$ | Horizontal | *Down* |
| $(-2, 0)$ | | Inflection Point |
| $(1, 0)$ | | Inflection Point |




3. Find each of the following limits, if it exists.

   (a) $\lim_{x\to\infty} \dfrac{5 - 3x^2}{1 + 2x^2} = \lim_{x\to\infty} \dfrac{-3x^2}{2x^2} = \lim_{x\to\infty} \dfrac{-3}{2} = \dfrac{-3}{2}$

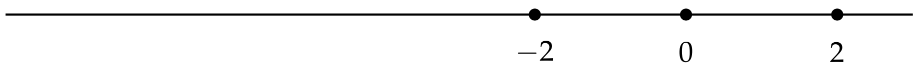
   (b) $\lim_{x\to\infty} \dfrac{1 + 4x - x^2}{x^3 - 5} = \lim_{x\to\infty} \dfrac{-x^2}{x^3} = \lim_{x\to\infty} \dfrac{-1}{x} = 0$

4. Suppose $f(x) = \dfrac{1}{x^2-4}$. Find the following:

(a) vertical asymptote(s) for the graph of $y = f(x)$
Denominator =0 $\implies x^2 - 4 = 0 \implies (x+2)(x-2) = 0 \implies x = 2\,; x = -2$
Both lines $x = 2$ and $x = -2$ are vertical asymptotes.

(b) horizontal asymptote for the graph of $y = f(x)$
$\lim\limits_{x\to\infty} \dfrac{1}{x^2-4} = \lim\limits_{x\to\infty} \dfrac{1}{x^2} = 0.$
Thus $y = 0$ is the horizontal asymptote.

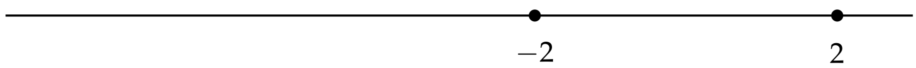
(c) interval(s) where $f(x)$ is increasing
$f'(x) = \dfrac{-2x}{(x^2-4)^2} = \dfrac{-2x}{(x+2)^2(x-2)^2}.$
Critical values are $0, 2, -2$.




| Test Value | -3 | -1 | 1 | 3 |
|---|---|---|---|---|
| Sign of $f'(x) = \dfrac{-2x}{(x-2)^2(x+2)^2}$ | + | + | − | − |
| $f$ is increasing/decreasing | ↗ | ↗ | ↘ | ↘ |

Therefore $f$ is increasing over the intervals $(-\infty, -2)$ and $(-2, 0)$.

(d) interval(s) where $f(x)$ is concave up.
$f''(x) = \dfrac{-2(x^2-4)^2 - (-2x)2(x^2-4)(2x)}{(x^2-4)^4} = \dfrac{2(x^2-4)[-x^2+4+4x^2]}{(x^2-4)^4} = \dfrac{2(3x^2+4)}{(x^2-4)^3}$
Inflection Points: $x = -2; x = 2$.




| Test Value | -3 | 1 | 3 |
|---|---|---|---|
| Sign of $f''(x) = \dfrac{2(3x^2+4)}{(x-2)^3(x+2)^3}$ | + | − | + |
| Concavity | Up | Down | Up |

Therefore $f$ is concave up over the intervals $(-\infty, -2)$ and $(2, \infty)$.

(e) Graph $y = f(x)$.