KEY **Applied Calculus – Quiz 8**

1. Given $x^5 + 2xy - y^3 = 2$, use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{dx^5}{dx} + \frac{d(2x)(y)}{dx} - \frac{dy^3}{dx} = \frac{d2}{dx}$$

$$5x^4 + \left[(y)\frac{d(2x)}{dx} + (2x)\frac{dy}{dx}\right] - \frac{dy^3}{dy}\cdot\frac{dy}{dx} = 0$$

$$5x^4 + y\cdot 2 + (2x)\frac{dy}{dx} - (3y^2)\frac{dy}{dx} = 0$$

$$(2x)\frac{dy}{dx} - (3y^2)\frac{dy}{dx} = -5x^4 - 2y$$

$$(2x - 3y^2)\frac{dy}{dx} = -5x^4 - 2y \Rightarrow \frac{dy}{dx} = \boxed{\frac{-5x^4 - 2y}{2x - 3y^2}}$$

2. The volume of a cantaloupe is given by $V = \frac{4}{3}\pi r^3$. The radius is growing at the rate of 0.25 cm/week at a time when the radius is 2 cm. How fast is the volume changing at that moment?

$$\frac{dV}{dt} = \frac{4}{3}\pi\frac{dr^3}{dt}$$

$$= \frac{4}{3}\pi\frac{dr^3}{dr}\cdot\frac{dr}{dt}$$

$$= \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$= \frac{4}{\cancel{3}}\pi(\cancel{3}\cdot 2^2)(0.25) = \boxed{4\pi \text{ cm}^3/\text{week}} \sim 12.56 \text{ cm}^3/\text{week}$$

3. Find the equation of the line tangent to the graph of $f(x) = 2e^{-3x}$ at the point $(0, 2)$.

$$f'(x) = 2\frac{de^{-3x}}{dx} = 2(-3)e^{-3x} = -6e^{-3x}$$

When $x = 0$, $m = f'(0) = -6e^0 = -6$

Point-Slope Form: $\boxed{y - 2 = -6(x - 0)}$

i.e. $y = -6x + 2$

4. Differentiate each of the following functions without simplifying your answers.

(a) $f(x) = x^4 e^{3x}$

$A \cdot B$

$$f'(x) = B\cdot A' + A\,B' = e^{3x}\cdot\left[\frac{dx^4}{dx}\right] + x^4\left[\frac{de^{3x}}{dx}\right]$$

$$= e^{3x}\cdot(4x^3) + x^4\cdot 3\cdot e^{3x}$$

$$= \boxed{4x^3 e^{3x} + 3x^4\cdot e^{3x}}$$

(b) $g(x) = \sqrt{e^{2x-1}} + e^{(1-3x)^5} = (e^{2x-1})^{\frac{1}{2}} + e^{(1-3x)^5}$

$$g'(x) = \frac{1}{2}(e^{2x-1})^{-\frac{1}{2}}\cdot(2)e^{2x-1} + 5(1-3x)^4(-3)e^{(1-3x)^5}$$