KEY

Applied Calculus – Quiz 9

1. Find the equation of the line tangent to the graph of $f(x) = \ln\left(\frac{2-x}{2x-1}\right)$ at the point $(1, 0)$.

$f(x) = \ln\left(\frac{2-x}{2x-1}\right) = \ln(2-x) - \ln(2x-1)$

$f'(x) = \frac{-1}{2-x} - \frac{2}{2x-1}$

$f'(1) = \frac{-1}{2-1} - \frac{2}{2-1} = -1-2 = -3$ → Point-Slope Form:

$y - 0 = 3(x-1)$

i.e. $y = 3x - 3$

2. Differentiate: $g(x) = \ln(1+x^3)^4 + (\ln 5x)^6 = 4\ln(1+x^3) + (\ln 5x)^6$

$g'(x) = \frac{d\,4\ln(1+x^3)}{dx} + \frac{d(\ln 5x)^6}{dx} = \frac{d\,4\ln(1+x^3)}{d(1+x^3)}\frac{d(1+x^3)}{dx} + \frac{d(\ln 5x)^6}{d(\ln 5x)}\cdot\frac{d\ln 5x}{d5x}\cdot\frac{d5x}{dx}$

$= 4\cdot\frac{1}{1+x^3}\cdot(3x^2) + \left[6(\ln 5x)^5\right]\cdot\frac{1}{5x}\cdot(5)$

$= \frac{12x^2}{1+x^3} + \frac{30}{x}(\ln 5x)^5$

3. Differentiate: $f(x) = \ln(e^{3x} + 4x) + \ln(\ln(5x))$

$f'(x) = \frac{d\ln(e^{3x}+4x)}{dx} + \frac{d\ln(\ln 5x)}{dx} = \frac{d\ln(e^{3x}+4x)}{d(e^{3x}+4x)}\frac{d(e^{3x}+4x)}{dx} + \frac{d\ln(\ln 5x)}{d(\ln 5x)}\cdot\frac{d\ln 5x}{d5x}\cdot\frac{d5x}{dx}$

$= \frac{1}{e^{3x}+4x}\cdot(3e^{3x}+4) + \frac{1}{\ln 5x}\cdot\frac{1}{5x}\cdot(5)$

$= \frac{3e^{3x}+4}{e^{3x}+4x} + \frac{1}{x\ln 5x}$

4. Compute $\int\left(\frac{6}{\sqrt[3]{x^2}} - \frac{e^{3x}}{5}\right)dx$

$= \int\left(6x^{-\frac{2}{3}} - \frac{1}{5}e^{3x}\right)dx$

$= 6\left(\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}\right) - \frac{1}{5}\cdot\frac{1}{3}e^{3x} + C = 18x^{\frac{1}{3}} - \frac{1}{15}e^{3x} + C$

5. Compute $\int \frac{6x^5 - 4x^3 + x^2 - 4}{x^3}\,dx = \int\left(\frac{6x^5}{x^3} - \frac{4x^3}{x^3} + \frac{x^2}{x^3} - \frac{4}{x^2}\right)dx$

$= \int\left(6x^2 - 4 + \frac{1}{x} - 4x^{-2}\right)dx$

$= \frac{6x^3}{3} - 4x + \ln|x| - 4\left(\frac{x^{-1}}{-1}\right) + C$

$= 2x^3 - 4x + \ln|x| + \frac{4}{x} + C$

6. If $f'(x) = 6x^3 - x$ and $f(1) = 3$, find $f(x)$.

$f(x) = \int(6x^3 - x)dx = 6\left(\frac{x^4}{4}\right) - \frac{x^2}{2} + C = \frac{3}{2}x^4 - \frac{1}{2}x^2 + C$

$f(1) = 3 = \frac{3}{2}(1)^4 - \frac{1}{2}(1)^2 + C = \frac{3}{2} - \frac{1}{2} + C = 1 + C$

Thus $C = 2$

$f(x) = \frac{3}{2}x^4 - \frac{1}{2}x^2 + 2$