

Chapter 8

Applications of Definite Integrals I: General Arguments

Here we look at many further quantities which give rise to antidifferentiation.

Most modern calculus textbooks contain numerous excellent examples of applications of definite integrals. Indeed, most examples which are likely to be seen in further studies of the physical sciences can trace back to calculus textbook-type problems.

Here we will make an attempt to accomplish the presentations of the usual topics, and others. We will also do the following:

1. Re-introduce the notion of infinitesimals into the physical analysis of these problems. This was a traditional approach which has fallen out of favor. While we will refer back to the Riemann sums for each case, that will be more for a “spot-check” of the reasoning behind the infinitesimals, and perhaps some proofs. However, the first introduction to most topics will be through infinitesimals. Besides, they make for prettier pictures!

Students are unlikely to be inspired by the Riemann sum proofs, which are often used to show that the quantity represented by the integral has the integrand as derivative. The proofs are technical, and leave a student to believe he would never come up with it himself. The differentials “cut to the chase,” and can be proved later after guessed, rather than derived from Riemann sums.

2. Finite Riemann sums used for numerical approximations of the quantities involved. This contains all the intuition (short of the proofs) contained in the most textbook developments of these integrals.
3. Explanations of why some guesses for the infinitesimals will not work.
4. Explanation of how to tell—by sight—if a particular differential is correct, and whether the Riemann sum form will actually converge to the desired quantity as the partition is refined.

8.1 Riemann Sums and Approximations of Cumulative Quantities

Suppose we had some data on the velocity of an object, and we wish to approximate its net displacement over a time interval. Suppose the data we have is the following:

8.2 Other Complications

Consider cases where the function is only piecewise continuous, and try to recover a continuous antiderivative. Consider, perhaps, some cases from electricity and magnetism regarding fields or potentials across interfaces.

Consider two “antiderivatives” of

$$f(x) = \begin{cases} 1, & x > 0, \\ -1, & x < 0. \end{cases}$$

One is continuous, the other not.

Can always recover a continuous antiderivative from a piecewise continuous function, so long as we don't have vertical asymptotes, for instance.